SOLVING QUADRATIC EQUATIONS

In this brush-up exercise we will review three different ways to solve a quadratic equation.

EXAMPLE 1: Solve: $6x^2 + x - 15 = 0$

SOLUTION

We check to see if we can factor and find that $6x^2 + x - 15 = 0$ in factored form is

$$(2x-3)(3x+5) = 0$$

We now apply the principle of zero products:

$$\begin{array}{r|r} 2x - 3 = 0 & 3x + 5 = 0 \\ 2x = 3 & 3x = -5 \\ x = \dfrac{3}{2} & x = -\dfrac{5}{3} \end{array}$$

We check each solution and see that $x = \frac{3}{2}$ and $x = -\frac{5}{3}$ are indeed solutions for the equation $6x^2 + x - 15 = 0$.

EXAMPLE 2: Solve: $4x^2 + 5x - 6 = 0$

SOLUTION

We can use the quadratic formula to solve this equation. This equation is in standard form, and

$$a = 4 \quad b = 5 \quad c = -6$$

We substitute these values into the quadratic formula and simplify, getting

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(5) \pm \sqrt{(5)^2 - 4(4)(-6)}}{2(4)} = \frac{-(5) \pm \sqrt{25 - (16)(-6)}}{8}$$

$$= \frac{-(5) \pm \sqrt{25 + 96}}{8} = \frac{-5 \pm \sqrt{121}}{8} = \frac{-5 \pm 11}{8}$$

We separate the two solutions and simplify.

$$x = \frac{-5 + 11}{8} \quad or \quad x = \frac{-5 - 11}{8}$$

$$= \frac{6}{8} = \frac{3}{4} \qquad\qquad = \frac{-16}{8} = -2$$

So $x = \frac{3}{4}$ and $x = -2$ when $4x^2 + 5x - 6 = 0$. Always check your results.

EXAMPLE 3: Solve by completing the square: $x^2 - x - 6 = 0$

SOLUTION

Step 1: This equation is in standard form. But we want the terms that contain the variable to be on the left and the constant to be on the right. So we add 6 to both sides, obtaining

$$x^2 - x = 6$$

The equation is now in the proper form for completing the square.

Step 2: Because *b* (the coefficient of *x*) is -1, $\frac{b}{2}$ is $\frac{-1}{2}$ and $\left(\frac{b}{2}\right)^2$ is $\left(\frac{-1}{2}\right)^2$

We add this value, $\left(\frac{-1}{2}\right)^2$, to both sides of the equation.

$$x^2 - x + \left(\frac{-1}{2}\right)^2 = 6 + \left(\frac{-1}{2}\right)^2$$

Step 3: We have transformed the left side of the equation into the square of the binomial $\left[x + \left(\frac{-1}{2}\right)\right]^2$. To see that this is so, multiply it out. You get

$$\begin{aligned}\left[x + \left(\frac{-1}{2}\right)\right]^2 &= x^2 + 2(x)\left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)^2 \\ &= x^2 + \left(\frac{-2x}{2}\right) + \left(\frac{-1}{2}\right)^2 \\ &= x^2 - x + \left(\frac{-1}{2}\right)^2\end{aligned}$$

We have "completed the square" of the left side.

We write the left side as $\left[x + \left(\frac{-1}{2}\right)\right]^2$ and simplify the right side, obtaining

$$\begin{aligned}\left[x + \left(\frac{-1}{2}\right)\right]^2 &= 6 + \left(\frac{-1}{2}\right)^2 \\ &= 6 + \frac{1}{4} \qquad \text{Squaring} \\ &= \frac{25}{4} \qquad \text{Combining and writing as an improper fraction}\end{aligned}$$

EXERCISES

1. $6n^2 + 30n - 36 = 0$
2. $y^2 + 11y + 18 = 0$
3. $x^2 + 13x + 40 = 0$
4. $16r^2 - 24r + 8 = 0$
5. $4r^2 + 16r - 180 = 0$
6. $3x^2 + 30x + 27 = 0$
7. $x^2 + 12x + 35 = 0$
8. $9x^2 - 15x - 6 = 0$
9. $2x^2 + 22x - 84 = 0$
10. $x^2 - 9x + 18 = 0$
11. $x^2 - 2x - 15 = 0$
12. $x^2 + 6x - 7 = 0$
13. $x^2 - 10x - 24 = 0$
14. $9x^2 - 18x + 8 = 0$
15. $4x^2 + 36x - 88 = 0$

SOLUTIONS

1. $n = -6$ and $n = 1$
2. $y = -2$ and $y = -9$
3. $x = -5$ and $x = -8$
4. $r = \frac{1}{2}$ and $r = 1$
5. $r = -9$ and $r = 5$
6. $x = -1$ and $x = -9$
7. $x = -5$ and $x = -7$
8. $x = -\frac{1}{3}$ and $x = 2$
9. $x = -14$ and $x = 3$
10. $x = 6$ and $x = 3$
11. $x = 5$ and $x = -3$
12. $x = 1$ and $x = -7$
13. $x = 12$ and $x = -2$
14. $x = 1\frac{1}{3}$ and $x = \frac{2}{3}$
15. $x = 2$ and $x = -11$